Optical spectrum of an electron gas under linearly polarized intense laser radiation

1998 J. Phys.: Condens. Matter 10 10787

(http://iopscience.iop.org/0953-8984/10/47/022)

J. Phys.: Condens. Matter **10** (1998) 10787–10793. Printed in the UK PII: S0953-8984(98)97169-7

Optical spectrum of an electron gas under linearly polarized intense laser radiation

W Xu†

Department of Engineering Physics, Institute for Superconducting and Electronic Materials, University of Wollongong, Wollongong, NSW 2522, Australia

Received 3 September 1998

Abstract. In this paper, I examine how a linearly polarized laser field affects the optical spectrum of an ideal three-dimensional electron gas (3DEG). The calculation is carried out on the basis of the random-phase-approximation dielectric function obtained recently. The dependence of the energy-loss function of a GaAs-based 3DEG on intense terahertz radiation, provided by, e.g., a free-electron laser field, has been investigated in detail.

1. Introduction

In 1995, powerful terahertz (10^{12} Hz or THz) or far-infrared (FIR) free-electron laser (FEL) radiation was successfully applied in investigation of nonlinear transport and optical properties in semiconductor-based electron gas systems [1, 2]. This has opened up a new field of research. From a fundamental point of view, *terahertz* is a very interesting frequency range lying between 'optical' and 'electrical' phenomena. THz-driven electron gas has therefore become a novel system to observe and study the photon-induced quantum effect. From a theoretical point of view, when a semiconductor-based electron gas is subjected to an intense THz laser field, the electron–photon interaction cannot be treated as a perturbation and approaches such as the high-frequency approximation may break down. Furthermore, due to the time-dependent nature of the laser field, one has to deal now with a time-dependent problem. As a result, the investigation of a THz-driven electron gas offers us a possibility to develop and examine time-dependent condensed matter theory on the basis of non-perturbative approaches.

Very recently, I have reported the result for the dielectric function of an ideal three-dimensional electron gas (3DEG) driven by a linearly polarized laser field [3]. In [3], the theoretical approach has gone beyond the conventional many-body theory in dealing with electron interactions with the radiation field in an electron gas system. (1) In the presence of a linearly polarized electromagnetic (EM) field, the single-electron Schrödinger equation can be solved exactly. (2) From the time-dependent electron wavefunction obtained from the solution of the Schrödinger equation, the retarded $G^+(\boldsymbol{K}; t > t')$ and advanced $G^-(\boldsymbol{K}; t > t')$ Green functions for electrons can be determined, where $\boldsymbol{K} = (k_x, k_y, k_z)$ is the electron wavevector. (3) From these Green functions, we can derive the electron density–density correlation function (or pair bubble) $\Pi(\boldsymbol{Q}; t > t')$ for a Fermi system at finite temperatures, where $\boldsymbol{Q} = (q_x, q_y, q_z)$ is the change of the

† E-mail address: wxu@wumpus.its.uow.edu.au.

electron wavevector. (4) Applying these results to the random-phase approximation (RPA) diagrams for effective electron–electron interaction, we can obtain the inverse RPA dielectric function $1/\epsilon(\boldsymbol{Q}; t > t')$ in t representation (i.e., time-space). (5) After first Fourier analysing $G^+(\boldsymbol{K}; t > t')$ and $1/\epsilon(\boldsymbol{Q}; t > t')$ according to the relative coordinates $\tau = t - t'$ then averaging the initial time t' in these quantities over a period of the radiation field, the electron density of states and the inverse RPA dielectric function in Ω representation (i.e., spectrum-space) can be obtained. In these steps, the effect of the radiation field is included exactly. In [3], I have studied the influence of the intense THz radiation on collective excitations such as plasmon modes in a 3DEG. From the fact that at present, optical studies have been popularly and widely used to measure the free electron density in inhomogeneous materials, to verify the dependence of the plasmon dispersion on electron density etc, it is of value to examine how a linearly polarized EM field affects the optical spectrum of a 3DEG, and this is the motivation of the present study. Below, I present analytical (see section 2) and numerical (see section 3) results obtained from this study.

2. Analytical results

In the present study, I limit myself to the case where the optical spectrum is induced by electronic transitions through RPA dielectric response. Other mechanisms, such as scattering by impurities and phonons, inhomogeneities, correlation etc, are not taken into account.

Let us consider the situation where a *driving* EM field with frequency ω and electric field strength F_0 is linearly polarized along the x-direction of a 3DEG and a weak *probing* field with frequency Ω is applied to measure the optical spectrum. In this case, the inverse RPA dielectric function in the Ω representation is given by [3]

$$\frac{1}{\epsilon(\boldsymbol{Q}, \Omega)} = \sum_{m=-\infty}^{\infty} \frac{J_m^2(r_0 q_x)}{1 - V_Q \Pi_0(Q, \Omega + m\omega)}. \tag{1a}$$

Here, $m > 0$ ($m < 0$) corresponds to m-photon absorption (emission) by electron interactions with the driving field, $J_m(x)$ is a Bessel function, $r_0 = eF_0/m^*\omega^2$ with m^* being the effective electron mass, $V_Q = 4\pi e^2/\kappa Q^2$ is the Fourier transform of the bare electron–electron interaction and κ is the dielectric constant of the material. Furthermore,

$$\Pi_0(Q, \Omega) = g_s \sum_{\boldsymbol{K}} \frac{f(E(\boldsymbol{K}) + E_{em}) - f(E(\boldsymbol{K} + \boldsymbol{Q}) + E_{em})}{\hbar\Omega + E(\boldsymbol{K}) - E(\boldsymbol{K} + \boldsymbol{Q}) + \mathrm{i}\delta} \tag{1b}$$

where $g_s = 2$ accounts for the spin degeneracy, $f(x)$ is the Fermi–Dirac function, $E(\boldsymbol{K}) = \hbar^2 K^2/2m^*$, $E_{em} = 2\gamma\hbar\omega$ is the energy of the driving EM field and $\gamma = (eF_0)^2/(8m^*\hbar\omega^3)$. In equation (1$b$), the blue-shift of the electronic energy spectrum by E_{em} is caused by the dynamical Franz–Keldysh effect [4]. It should be noted that for studying a dynamical quantity such as the optical spectrum, Ω in equation (1a) can be taken as the frequency of the probing field.

It is well known that the optical spectrum of an electron gas system can be obtained theoretically from the imaginary part of the inverse dielectric function, $-\mathrm{Im}\,\epsilon^{-1}(\Omega)$, which measures the energy-loss of fast electrons via absorption of the probing field with frequency Ω [5]. From equation (1), the averaged imaginary part of the inverse dielectric function (or so called energy-loss function) is obtained as

$$-\mathrm{Im}\,\epsilon^{-1}(\Omega) = \frac{-1}{n_e} \sum_{\boldsymbol{Q}} \mathrm{Im}\,\frac{1}{\epsilon(\boldsymbol{Q}, \Omega)} = -\frac{1}{2}\int_0^\pi \mathrm{d}\theta\,\mathrm{Im}\,\epsilon^{-1}(\Omega, \theta)\sin\theta \tag{2a}$$

where n_e is the electron density of the 3DEG system, θ is the polar angle to the x-axis along which the driving EM field is polarized, and the angular distribution of the energy-loss function is

$$-\mathrm{Im}\,\epsilon^{-1}(\Omega,\theta) = -\sum_{m=-\infty}^{\infty} \mathrm{Im}\,\epsilon_m^{-1}(\Omega,\theta) \tag{2b}$$

which reflects the fact that the optical spectrum comes from all possible photon processes via electron interactions with the driving EM field. The contribution due to the m-photon process to the energy-loss function is given by

$$-\mathrm{Im}\,\epsilon_m^{-1}(\Omega,\theta) = \frac{-1}{2\pi^2 n_e}\int_0^\infty \frac{\mathrm{d}Q\,Q^2 J_m^2(r_0 Q\cos\theta)[V_Q\,\mathrm{Im}\,\Pi_0(Q,\Omega+m\omega)]}{[1-V_Q\,\mathrm{Re}\,\Pi_0(Q,\Omega+m\omega)]^2+[V_Q\,\mathrm{Im}\,\Pi_0(Q,\Omega+m\omega)]^2}. \tag{2c}$$

From equation (2), we see that in the presence of the linearly polarized driving EM field, the optical spectrum of an electron gas is anisotropic, i.e., it depends on the angle θ. The physical reason behind this is that the linearly polarized driving EM field can break the symmetry of the sample geometry. We note that the change of the electron wavevector along the direction where the driving EM field is polarized (i.e., q_x) plays a role in switching different photon processes. For example, when $q_x \to 0$ only the 0-photon process contributes to the optical spectrum, due to $J_m^2(0) = \delta_{m,0}$. With increasing $|q_x|$, entailing $J_m(r_0 q_x) \neq 0$, other photon processes including multiphoton channels can contribute to the energy-loss function. Moreover, for high-frequency $\omega \gg 1$ and/or low-intensity $F_0 \ll 1$ driving fields so that $r_0 \sim 0$ and $\gamma \sim 0$, the optical spectrum given by equation (2) becomes the well known result obtained in the absence of the driving EM field and $\mathrm{Im}\,\epsilon^{-1}(\Omega,\theta) = \mathrm{Im}\,\epsilon^{-1}(\Omega)$ does not depend on the angle θ.

In the low-temperature limit (i.e., $T \to 0$) where $f(x) \to \Theta(E_F - x)$ with E_F being the Fermi energy, the energy-loss function equation (2c) becomes

$$\begin{aligned}-\mathrm{Im}\,\epsilon_m^{-1}(\Omega,\theta) = {}& \frac{e^2 m^{*2}}{8\pi^2\kappa\hbar^4 n_e}\Theta(E_F - E_{em})\bigg[\Theta[E_F - E_{em} - \hbar(\Omega+m\omega)]\int_{a_-}^{a_+}\mathrm{d}x\, x^3 \\ & \times \frac{(a_+ - x)(x - a_-)}{P(x,\Omega+m\omega)} J_m^2\big(r_0\sqrt{2m^* x/\hbar^2}\cos\theta\big) - \Theta[E_F - E_{em} + \hbar(\Omega+m\omega)] \\ & \times \int_{b_-}^{b_+}\mathrm{d}x\, x^3 \frac{(b_+ - x)(x - b_-)}{P(x,\Omega+m\omega)} J_m^2\big(r_0\sqrt{2m^* x/\hbar^2}\cos\theta\big)\bigg]. \end{aligned} \tag{3}$$

Here,

$$a_\pm = [\sqrt{E_F - E_{em}} \pm \sqrt{E_F - E_{em} - \hbar(\Omega+m\omega)}]^2 \tag{4a}$$

$$b_\pm = [\sqrt{E_F - E_{em}} \pm \sqrt{E_F - E_{em} + \hbar(\Omega+m\omega)}]^2 \tag{4b}$$

$$P(x,\Omega) = \left[x^{5/2}\frac{e^2(2m^*/\hbar^2)^{1/2}}{8\pi\kappa}R(x,\Omega)\right]^2 + \left[\frac{e^2(2m^*/\hbar^2)^{1/2}}{8\kappa}I(x,\Omega)\right]^2 \tag{4c}$$

$$\begin{aligned}R(x,\Omega) = {}& 8x^{3/2}\sqrt{E_F - E_{em}} + [4x(E_F - E_{em}) - (x+\hbar\Omega)^2] \\ & \times \ln\left|\frac{2\sqrt{x(E_F - E_{em})} + x + \hbar\Omega}{2\sqrt{x(E_F - E_{em})} - x - \hbar\Omega}\right| + [4x(E_F - E_{em}) - (x - \hbar\Omega)^2] \\ & \times \ln\left|\frac{2\sqrt{x(E_F - E_{em})} + x - \hbar\Omega}{2\sqrt{x(E_F - E_{em})} - x + \hbar\Omega}\right| \end{aligned} \tag{4d}$$

and

$$I(x, \Omega + m\omega) = \Theta[E_F - E_{em} - \hbar(\Omega + m\omega)]\Theta[(a_+ - x)(x - a_-)][(a_+ - x)(x - a_-)]$$
$$-\Theta[E_F - E_{em} + \hbar(\Omega + m\omega)]\Theta[(b_+ - x)(x - b_-)][(b_+ - x)(x - b_-)]. \quad (4e)$$

Equation (3) indicates that in the low-temperature limit, the optical spectrum of an electron gas driven by EM radiation field can only be observed when the Fermi energy of the system E_F is above the energy of the driving radiation field E_{em}. This can be understood by the fact that in the presence of the driving field, the dielectric response to the probing field, via electronic transitions accompanied by absorption of the probing and driving fields, can only occur when $E_F > E_{em}$. When $E_F < E_{em}$, the emission of photons by electrons will be the principal channel for relaxation of excited electrons in the system.

3. Numerical results

The numerical results of this paper pertain to GaAs-based 3DEG structures. For GaAs, the effective electron mass is $m^* = 0.0665m_e$, with m_e being the rest electron mass, and the dielectric constant is $\kappa = 12.9$. In the calculations, I consider n-type-doped GaAs with the typical electron density $n_e = 10^{17}$ cm^{-3}. Furthermore, I take THz FEL radiation with, typically, the frequency $\omega/2\pi \sim 1$ THz and intensity $F_0 \sim 10$ kV cm^{-1} as the source of the driving EM field. Normally, the FELs provide linearly polarized EM radiation fields.

The influence of the linearly polarized THz laser radiation on Fermi energy in a GaAs-based 3DEG system has been discussed in [3] (see figures 4 and 5 there). (1) Under low-frequency and/or high-intensity radiations, when $E_{em} \gg \hbar\omega$, the Fermi energy E_F is mainly determined by the energy of the radiation field E_{em} via the dynamical Franz–Keldysh effect. In this case, $E_F > E_{em}$ and, therefore, the optical spectrum via dielectric response can be observed. (2) In an intermediate radiation frequency and intensity regime, E_F is determined mainly by photon emission processes and, consequently, $E_F < E_{em}$. In this case, the low-temperature optical spectrum cannot be observed and this regime of F_0 and ω is a window for propagation of EM waves. (3) For relatively high-frequency and/or low-intensity radiations, when $E_{em} \sim \hbar\omega$, E_F is determined mainly by 0-photon and photon absorption processes and, so, $E_F > E_{em}$. In this case, $E_F - E_{em}$ increases with increasing ω and/or decreasing F_0 and the optical spectrum can be observed. (4) For $\omega \gg 1$ and/or $F_0 \ll 1$, entailing $E_{em} \to 0$ and $r_0 \to 0$, the radiation affects very weakly the Fermi energy and the optical spectrum tends to that observed in the absence of the laser field. Below, I discuss the influence of the intense THz driving field on optical spectrum in the case of (3).

The energy-loss function at a fixed driving field for different θ angles is shown in figure 1, where the angular dependence of the optical spectrum is evident. The variation of θ corresponds to the change of the possibility for electronic transitions achieved by change of the electron wavevector (or momentum) along the direction where the driving field is polarized. The variation of q_x will result in different processes of photon emission and absorption by electron interactions with the driving field. Therefore, the optical spectrum induced by dielectric response in the presence of an intense driving EM field depends on the angle θ. $\theta = 90^\circ$ implies $q_x = 0$ and so only the 0-photon process gives rise to the energy-loss function. $\theta = 0$ corresponds to $q_x = Q$ where the strongest effect of the driving field can be measured. The results obtained from further numerical calculations show that a stronger anisotropic feature of the optical spectrum can be observed at a driving field with higher radiation intensity and/or lower radiation frequency, because $r_0 = eF_0/m^*\omega^2$ (see equation (3)).

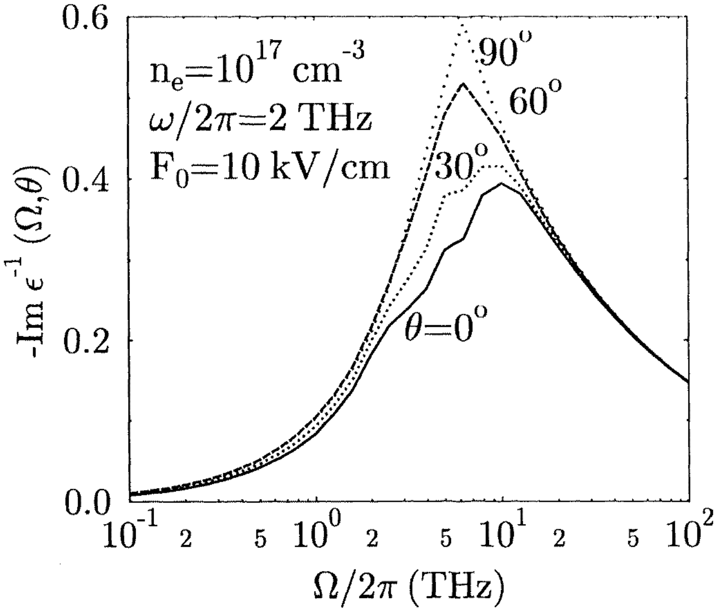

Figure 1. Energy-loss function at a fixed driving field (with frequency ω and intensity F_0) for different θ angles. θ is the polar angle to the x-axis along which the driving laser field is polarized, n_e is the electron density of the 3DEG and Ω is the frequency of the probing field.

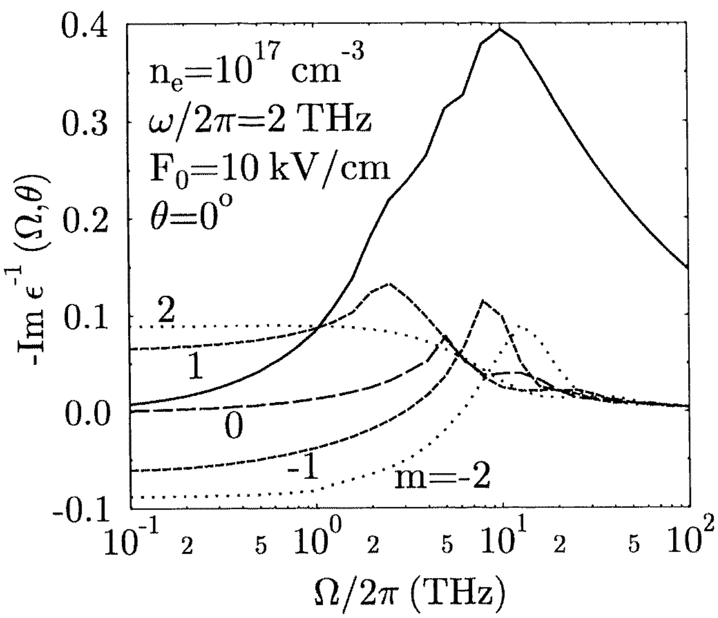

Figure 2. Contribution due to different optical processes to the energy-loss function at $\theta = 0^\circ$ for a fixed driving field. Here $m > 0$ and $m < 0$ correspond, respectively, to the processes of photon absorption and emission by electron interactions with the driving field. The solid curve is the total energy-loss function.

The contribution from different optical processes due to electron interactions with the driving field to the energy-loss function is shown in figure 2 at a fixed driving field and at $\theta = 0$. In the presence of an intense driving EM field, the electronic transitions can be achieved by emission ($m < 0$) and absorption ($m > 0$) of photons, including multiphoton ($|m| > 1$) processes. These electronic transition events can be observed by measuring the optical spectrum via dielectric response, as can be seen in figure 2. From figure 2, we note that for low-frequency probing fields, the energy-loss function caused by photon emission processes is negative, which implies an optical gain due to the interaction between electrons and the driving field. The results obtained from further calculations indicate that with increasing F_0 and/or decreasing ω of the driving field, a stronger effect of photon emission can be observed in the optical spectrum.

Figure 3 shows the energy-loss function at a fixed ω and at $\theta = 0$ for different intensities of the driving fields. With increasing F_0, the energy-loss function decreases. This is due to the following two factors. (1) At $\omega/2\pi = 2$ THz and when $F_0 \leqslant 15$ kV cm^{-1}, $E_F - E_{em}$ decreases with increasing F_0. A smaller $E_F - E_{em}$ implies fewer channels for dielectric response accompanied by absorption of the driving and probing fields. (2) With increasing F_0, the electronic transitions via photon emission increase, which makes a negative contribution to the energy-loss function in some probing field regimes.

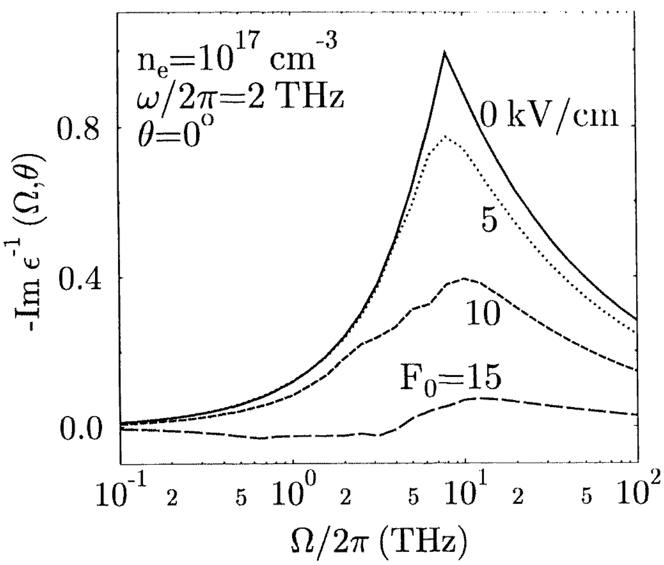

Figure 3. Influence of intensity of the driving field F_0 on energy-loss function at a fixed radiation frequency ω for $\theta = 0°$. $F_0 = 0$ is the case in the absence of the driving field.

4. Further notes

The theoretical results shown in this paper indicate that the influence of a driving EM field on the optical spectrum via dielectric response in a 3DEG can be achieved mainly through two important parameters: $r_0 = eF_0/m^*\omega^2$ and $\gamma = (eF_0)^2/(8m^*\hbar\omega^3)$. These parameters connect to the frequency and intensity of the driving field and to the material parameter such as effective electron mass of the sample system. For a GaAs-based 3DEG structure subjected to an EM driving field with $\omega/2\pi \sim 1$ THz and $F_0 \sim 10$ kV cm^{-1}, the conditions such as $r_0 q_x \sim 1$ and $\gamma \sim 1$ (so, $E_{em} = 2\gamma\hbar\omega \sim \hbar\omega$) can be satisfied. As a consequence, (i) the electron kinetic energy and the Fermi energy of the system are comparable to the THz photon energy and to that of the radiation field; (ii) the driving field will couple strongly to the 3DEG system and will vary significantly the electronic structure of the system and (iii) the momentum and energy relaxation via electronic transitions induced by, e.g., dielectric response will be strongly modified by the driving EM field.

It should be noted that such a radiation condition has been realized by the THz free-electron laser sources developed recently in, e.g., UCSB (USA) [1] and FELIX (The Netherlands) [2]. Moreover, when a GaAs-based electron gas system is subjected to THz FEL radiation, the blue-shift of the energy spectrum by $E_{em} = (eF_0)^2/(4m^*\omega^2)$ via the dynamical Franz–Keldysh effect has been successfully observed in a very recent experimental measurement [6]. It is therefore hoped that the phenomena discussed in this paper, such as the anisotropic feature of the optical spectrum (figure 1), the dependence of the energy-loss function on intensity of the THz laser field (figure 3), the multiphoton effect (figure 2), the window effect etc, will be verified experimentally.

Acknowledgment

This work was supported by the Australian Research Council.

References

[1] Asmar N G, Markelz A G, Gwinn E G, Černe J, Sherwin M S, Campman K L, Hopkins P F and Gossard A C 1995 *Phys. Rev.* B **51** 18 041
[2] Langerak C J G M, Murdin B N, Cole B E, Chamberlain J M, Henini M, Pate M and Hill G 1995 *Appl. Phys. Lett.* **67** 3453
[3] Xu W 1998 *Phys. Rev.* B **57** 15 282
[4] Jauho A P and Johnsen K 1997 *Phys. Rev. Lett.* **76** 4576
Bertoncini R and Jauho A P 1991 *Phys. Rev.* B **44** 3655
[5] See, e.g., Glicksman M 1971 *Solid State Physics* vol 26 (New York: Academic) p 275
[6] Nordstrom K B, Johnsen K, Allen S J, Jauho A-P, Birnir B, Kono J, Noda T, Akiyama H and Sakaki H 1998 *Phys. Rev. Lett.* **81** 457